J. L. Berggren. *Episodes in the Mathematics of Medieval Islam*. 2^{nd} Edition. New York: Springer, 2016. xii + 256 pages. ISBN: 9781493937806.

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J. L. Berggren's *Episodes in the Mathematics of Medieval Islam* has remained the only reliable English introduction to our field since the first edition appeared in 1985. It is unfortunate that the flurry of research produced during the past thirty years has not prompted others to take up the task of writing books on this topic that are accessible to a general audience or undergraduate students. I had a plan to write one myself, but it is now hopelessly far down the list of things I 'need' to do. Berggren should also be applauded for turning his attention away from specialist studies by issuing a new edition of *Episodes*.

Readers of *Nazariyat* need only be reminded of the book's organization. The introduction gives, along with other preliminaries, the biographies of four prominent Muslim scientists: al-Khwārizmī (d. after 232/847), al-Bīrūnī (d. after 442/1050), 'Umar al-Khayyām (d. ca. 526/1131), and al-Kāshī (d. 832/1429). Chapters 2 through 6 cover arithmetic, geometry, algebra, trigonometry, and spherical trigonometry, respectively. Chapter 7, on number theory and combinatorics, is a new chapter. An index rounds out the book. Exercises and a bibliography are given at the end of each chapter. Rather than provide sweeping overviews of all known accomplishments in each topic, Berggren takes the reader through the nuts and bolts of the calculations and constructions of samples of the mathematics from selected texts. The exercises at the end of each chapter should ensure that readers—not just students—take an active part in all of this. After all, one learns math by *doing it*.

Only a few changes have been made to the chapter sections taken over from the 1985 edition. What makes this book a second edition is that Berggren has added new sections to most chapters and an entirely new seventh chapter. He explains in the Preface that in addition to taking into account texts and studies that have

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been published during the past three decades, he addresses a shortcoming of the first edition by covering episodes from the mathematics of the western part of the Islamic world (vii). Three sections from the first edition have been heavily revised: Chapter 1.2. "Islam's acquisition of foreign science"; the opening paragraphs to Chapter 3. "Geometry in the Islamic World"; and Chapter 6.4. "Stereographic projection and the astrolabe."

Here is a list of new sections:

2.3. "The Arithmetic of Common Fractions": Berggren reviews how Arabic arithmeticians wrote and calculated with fractions in the western part of the Islamic world.

3.7. "A Problem in Geometrical Optics": This section covers a couple lemmas for the solution to Alhazen's (ca. 432/1040) problem as related in al-Mu'taman b. Hūd's (d. 478/1085) *Book of Completion*, written in al-Andalus.

3.9. "Al-Mu'taman b. Hūd's Book of Completion": His proof of Ceva's Theorem from the same book was not known from Arabic sources until recently.

3.10. "Practical Geometry of Measurement": Berggren surveys various problems from Muḥammad ibn ʿAbdūn's (d. after 366/976) *On Measurement*, written in Cordova.

4.7. "Al-Samaw'al on the Table of Binomial Coefficients": Some of the lemmas, with proofs, are described for determining coefficients of terms in the expansion of $(a+b)^n$ that al-Samaw'al (d. ca. 571/1175) cites from a lost work by al-Karajī (d. ca. 5th /11th century).

4.8. "Algebra in the Maghrib": Some elements of Ibn al-Bannā''s (d. 721/1321) book on algebra, written in Morocco during the late 13th century, are discussed.

4.8.1. "Ibn al-Bannā' on Quadratic Equations": Ibn al-Bannā' proved the rules for solving simplified equations not by geometry, but by completing the square in the context of the equation.

4.8.2. "Algebraic Notation in the Maghrib": Berggren gives examples of the notation for polynomials in the works of two western algebraists, Aḥmad ibn Qunfūdh (d. 810/1407) and Aḥmad al-Qaṭrawānī (d. 8th/late 14th or early 15th century).

5.3. "The Seventh Trigonometric Function": Calculations with the versed sine are discussed in connection with the work of Abū al-Ḥasan al-Marrakūshī (d. ca. 660/1262).

Chapter 7: "Number Theory and Combinatorics in the Islamic World"

7.1. "Number Theory": The chapter begins with a brief discussion of Greek origins and definitions of perfect and amicable numbers.

7.1.1 "Representing Rational Numbers as Sums of Squares": This section gives Ibn al-Bannā'''s treatment of the indeterminate problem of decomposing a number into two squares.

7.1.2 "Figured Numbers": Berggren mentions Greek and Indian works on figured (sometimes called 'figurate') numbers and then outlines Ibn Mun'im's approach to showing that any figured number can be expressed with triangular numbers. Ibn Mun'im (d. ca. 625/1228) worked in the Maghreb.

7.1.3. "Magic Squares": Abū al-Wafā'''s procedure for constructing a 5 by 5 bordered magic square is explained.

7.2 "Combinatorics": This section begins with a brief mention of work done in India, Greece, and the early years of Islam, with special mention of Thābit ibn Qurra (d. 288/901).

7.2.1 "Enumerating Words of *k* Distinct Letters in an Alphabet of *n* Letters": Ibn al-Bannā'''s solution for calculating *n* things taken *k* at a time is explained.

7.2.2 "Ibn Mun'im on Counting Arabic Words of at Most Ten Letters": Ibn Mun'im performed a similar calculation, by calculating the number of possible words that conform to the rules of Arabic grammar.

7.2.3 "Ibn al-Majdī on Enumerating Polynomial Equations": From al-Khwārizmī and al-Khayyām we know that there are six types of polynomial equation of degree two or less and twenty-five equations of degree three or less. The Egyptian astronomer and mathematician Ibn al-Majdī (d. 850/1447) calculates that there are 90 equation types of degree four or less and outlines rules for solving the general case.

Berggren's descriptions of the mathematics in each section are thorough and helpful. He typically situates the work in question in its historical context and explains possible points of confusion as he walks the reader through the steps. Also, he notes in many instances that his modern notation is written rhetorically in the original. The use of modern notation to expose readers to medieval mathematics for the first time is appropriate when it is made clear.

The only place in the book that I find in need of correction is in Chapter 8.2, Berggren's account of the Arabic algebraic notation. He writes: "After the work of Ibn al-Bannā' one finds the widespread use of a shorthand algebraic notation in the

Maghrib, which replaced the purely rhetorical algebra that was so common in Eastern Islam" (145). Notation did not replace the rhetorical form of algebra; rather, it was used to perform calculations on a dust-board or other erasable surface, and rhetorical versions were composed for books. In fact, notation is shown in books only to illustrate what is to be written on the board.[1] This algebraic notation was developed from calculations with Arabic numerals, which were also intended for the dust-board. Berggren explains the difference between dust-board notation and rhetorical text in this context (34).

Berggren remarks of a commentary of Ibn al-Qunfūdh that "[s]ince he does not comment on his use of the notation it seems probable that the use of such notation predates him" (145). In fact, notation also appears close to two centuries earlier in Ibn al-Yāsamīn's (d. 601/1204) *Grafting of Opinions of the Work on Dust-Figures*. Because he also makes no comment on this notation, it must have been circulating in the Maghreb at least as early as the late twelfth century.[2] And last, Berggren identifies the sign that functions as a 'minus' as "the Arabic word *lā*". It is, instead, a truncated *illā* ("less") (146).

The only other correction I would suggest—and this one is minor—concerns an Arabic fraction, which Berggren translates as "four ninths and five times an eighth of a ninth and one half of an eighth of a ninth" (39). The "five times an eighth of a ninth" should be simply "five eighths of a ninth". In addition, there are some typesetting problems with the second edition, mainly quotations that should have been set apart (indented) from the main text with a smaller font size (125, 131, 136, 152, 191, 207, and 213).

Many photos that were in black and white in the first edition are now shown in color, while several diagrams are a little less clear perhaps because they are scans of the 1985 originals. They are still clear enough, however. Many of the new photos show postage stamps honoring medieval Muslim scientists.

Overall, Berggren has done a fine job of updating and expanding his 1985 classic. Now others among us need to begin writing for a wider audience too.

1 For its detailed account, see Jeffrey A. Oaks, "Algebraic symbolism in medieval Arabic," Philosophica 87 (2012): 27-83.

2 This notation is shown in T. Zemouli's edition of Ibn al-Yāsamīn's book. T. Zemouli, "Mu'allafāt Ibn al-Yāsamīn al-Riyāḍiyya [Mathematical Writings of Ibn al-Yāsamīn]" (M.Sc. Thesis in History of Mathematics, Ecole Normale Supérieur en Lettres et Sciences (ENS), Algiers, 1993), and is described in Ahmed Djebbar. *L'Algèbre Arabe: Genèse d'un Art* (Paris: Vuibert, 2005), 92; Oaks, "Algebraic symbolism in medieval Arabic"; Driss Lamrabet, *Introduction à l'Histoire des Mathématiques Maghrébines*, 2nd ed. (Rabat: Driss Lamrabet, 2014), 347ff, among other sources.