# N A Z A R İ Y A T

Analyzing a Text Within its Period: Why Did We Misunderstand Masdariyecizāde Hüseyin Efendi's Treatise on *Teslīs-i Zāviye and Kavs?* 

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**Abstract:** Masdariyecizāde Hüseyin Efendi, one of the lecturers at Mühendishāne-i Berrī-i Hümāyūn (The Imperial School of Military Engineering), wrote a treatise in h.1238/c.1822 on the ancient problem of trisecting an angle smaller than ninety degrees using Euclidean geometry. This famous problem has its roots in the ancient Greek. The treatise contains an assumed solution to the problem by using tools of Euclidean geometry, a straightedge (an unmarked ruler) and a compass. The proof is recorded under the signatures indicating the approval of the engineering faculty. The available academic literature on this treatise generally contains comments denigrating the work, the author and the scholarly environment of the period based on the claim that it had already been proven at the time Hüseyin Efendi published his treatise, that a positive result could not be reached with the limited tools used in the solution. All this contemporary research is originated from a sole source, Salih Zeki Bey's articles on the subject written a century ago, the accuracy of which is debatable in terms of its contents. This study focuses on the claims based on this common source as well as the history of the solution to the problem, and thus provides a correction to the erroneous information on the subject.

**Keywords:** Trisecting the angle, tes<u>l</u>īs<u>-</u>i zāviye, Masdariyecizāde Ḥüseyin Efendi, Mühendishāne, Salih Zeki Bey

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# Background

The problem of trisecting the angle smaller than a right angle is one of the most wellknown geometry problems of antiquity, along with doubling the volume of a cube, drawing a square equal to the area of a circle, and drawing a regular heptagon. The rule for these problems is that the solution should be obtained by using only the two tools of Euclidean geometry, a straightedge and a compass. A straightedge is a ruler that does not have any measurement marks on it, but only allowing one to draw a straight line. The compass is used to draw a circle with a radius of a given length.

The names of these problems are still remembered for more than two thousand years as they occupied the minds and time of many famous mathematicians in every period and civilization until it was concluded that their solutions were impossible. Some mathematicians tried to find evidence for the impossibility of solutions, while others pursued a positive result. We now know that solutions to these problems are impossible under the given conditions, but it was only in the middle of the nineteenth century that mankind reached this conclusion. The story of a two thousand years old problem is quite long. The historical process of the problem of trisecting the angle and other problems in the East and West can be found in many popular and academic publications. The works around which this study will shape are Masdariyecizāde Ḥüseyin Efendi's treatise titled Teslīs-i Zāviye Mes'elesi" (The Problem of Teslīs-i Zāviye), published in the *Resimli Gazete* in j.1307/c.1891.

# Maşdāriyecizāde Seyyid Hüseyin Efendi and Teslīs-i Zāviye and Kavs

There is not sufficient information about the life of Masdariyecizāde Seyyid Hüseyin Efendi (d. around h.1240/c.1825). We learn some little information from the preface of his treatise that he served as a high rank lecturer (serhalīfe) at Mühendishāne-i Berrī-i Hümāyūn. He taught at Mühendishāne for more than thirty years and was promoted to the fifth senior lecturer.<sup>1</sup> From the few documents in the State Archives, we learn that he was involved in various tasks in the army and reinforcing military fortifications from time to time together with other lecturers

<sup>1</sup> Ekmeleddin İhsanoğlu vd., Osmanlı Matematik Literatürü Tarihi 1 (İstanbul: IRCICA, 1999), 273.

and engineers of Muhendishāne. The efforts to reinforce the castles of Silistra, Nicopolis, Turnu, Brăila, and the Sulina Walls in the Balkans were among these tasks.<sup>2</sup> His known mathematical works include *Mesāha ve Müsellesāt* (Measuring and Triangles) and the treatises titled *Teslīs-i Zāviye ve Ķavs* (Trisecting the Angle and Arc). The latter will be the subject of this study.

The note recorded at the end of the treatise of *Teslīs\_i Zāviye and Ķavs*, reads "tamma ṭab' hādhihi al-risāla bi-'awn rabb al-bariyye bi-ma'rifat Ibrāhīm Ṣā'ib mudīr-i Dār al-Ṭibā'a fī awṣāṭ Rabī' al-awwal li-sana thamān wa thalāthīn wa mi'atayn wa alf"<sup>3</sup>. As we understand from this, the printing was completed at Dāru't-Ṭibā'a in the middle of the month of Rabī' al-avval in h.1238. The date corresponds to the end of the year c.1822.

In the preface, after the classical words of hamdala and salvala, Hüseyin Efendi begins to explain the reason for writing his work. He states that the problem of trisecting a plane angle or an arc segment with the help of geometry (i.e. only lines and circular curves are to be used in the solution) is a well-known problem that has been studied by philosophers, astronomers, geographers and engineers as well as mathematicians since ancient times. Moreover, he is aware that *Encyclopédie*, the celebrated encyclopedic work published by Jean Le Rond D'Alambert (d.1783) and Denis Diderot (d. 1784), along with many other geometry books, states that this problem is unsolvable.

"... Ma'lūm ola ki fünūn-i ḥikemiyye ve 'ulūm-i riyāżiyye asḥābından gerek ḥükemā-i mütekaddimīn ve ḥükemā-i müte'ahhirīn ve gerek ehl-i hey'et ve ehl-i coğrafya ve gerek bi'l-cümle milel-i āhar mühendisleri beynlerinde devr-i Ādem'den bu tārīhe gelince aranılagelub hendese ţarīkiyle bir zāviye-i musaṭṭaḥayı yāhūd bir kavsı mütesāviyeten üç cüz'e taksīm itmek bulunamamış oldığı mütevātir ve meşhūr oldığından ekser hendese kitāblarında 'adīmü'l-imkān deyu taḥrīr ve tasṭīr itmeleriyle ve hālen Avrupa düvelinig beyninde karīb 'aṣırda cemī ' 'ulūm-i fünūnu şāmil te'līf iyledikleri Enciklopedya [Encyclopédie] nām kitābıg 'ulūm-i ta 'līmiyyesinig cild-i evvelinde taṣrīḥ olundığı üzere muţlakan bir zāviyenig yāhūd bir kavsıg mütesāviyeten üç cüz'e taksīmi ve bir muka 'abıg dı 'fına müsāvī muka 'ab-ı āhar inşāsı ve bir dā'ireye müsāvī bir murabba ' resmi bi'l-hendese ilā yümnā hāzā gelen mühendisīn bulamadıklarını tasrīḥ etmeğle..."

<sup>2</sup> Devlet Arşivleri Başkanlığı Osmanlı Arşivi (OA) Cevdet Askeriye (C.AS) 590/24845 ve 670/28165; OA, Cevdet Nafia (C.NF) 14/692; OA, Cevdet Maarif (C.MF) 33, 1641.

<sup>3</sup> Hüseyin Efendi, Teslîs-i Zâviye ve Kavs, (İstanbul: Dârü't-Tıbâ'a, h.1238), 34.

"It is well known that the scholars of the philosophical sciences and the mathematical sciences, both the earlier scientists and the later scientists, as well as the astronomers and geographers, or mathematicians of all other nations have been sought for a solution to the problem of trisecting the plane angle or arc by means of geometry, and it is famous and told by everyone that a solution could not be found since the time of Adam to the present day. As it is written in most of the geometry books, the solution is not possible, and as it is explained in the part of learned sciences of the first volume of the book called *Encyclopedia*, which still contains all the knowledge and sciences among the European states, that the mathematicians could not definitively find the solution until now for the problems of trisecting the angle or the arc, doubling the cube and squaring the circle..."<sup>4</sup>

After mentioning this worldwide search and thus the importance of the problem, Hüseyin Efendi adds that a solution to the problem has been sought for thirty years at Muhendishāne, and even *Usūl-i Hendese*, the Turkish translation of the *Elements* prepared by the Head Lecturer (Başhoca) Hüseyin Rıfkı Tamānī, states that it is not possible to solve the problem:

"...ve otuz seneden beru Mühendishāne-yi Hümāyūn'da dahi bu mevādd-ı selāseden teslīs-i zāviye māddesi aranılagelub ve lisān-ı Türkī üzere Uklīdis tercümesi olan Usūl-i Hendese kitābında dahi üçüncü makālesinig yirmi üçüncü da 'vā-yı 'amelīsinig tenbīhinde muţlakan dā'ireden bir kavsı hendese-i ma 'lūme ţarīki üzere mütesāviyeten üç cüz'e taksīm itmek a 'dīmü'l-imkāndır deyu muşarrah ve mesţūr iken ve bu māddenig hendese ţarīkiyle 'adīmü'l-imkān oldığı beyne'l-hükemā ve beyne'l-mühendisīn meşhūr ve mütevātir ise de..."

"For thirty years, a way of trisecting the angle has been searched for at Muhendishāne-i Humāyūn. Even in the Turkish translation of Euclid, *Usūl-i Hendese*, in remark of the twenty-third theorem of the third article, it is clearly said and written that it is not possible to trisect an arc from a circle with a known geometrical way. However, it is still popular and controversial among scientists and mathematicians that the solution of this problem is impossible in geometry..."<sup>5</sup>

Obviously, there was still an expectation that the problem could be solved positively since these claims of impossibility were not based on evidence. As a matter of fact, Sayyid Hüseyin Efendi says that he was able to solve this supposedly impossible problem thanks to the help of Allah, the miracle of the Prophet's

<sup>4</sup> Hüseyin Efendi, *Teslîs-i Zâviye ve Kavs*, 3-4.

<sup>5</sup> Hüseyin Efendi, *Teslîs-i Zâviye ve Kavs*, 4-5. *Usûl-i Hendese*'de ilgili kısım için bkz. Hüseyin Rıfkı Tamânî, *Usûl-i Hendese*, (İstanbul: Mekteb-i Harbiye-i Şahane Matbaası, h.1269), 89.

prophecy and the effect of the blessing of the justice of Sultan Mahmūd II, who was on the throne at that time:

"...ḥamden summe hamden Cenāb-ı Ḥakīm-i Muṭlak-ı Vācib Te ʿālanıg 'ināyātı ve dū 'ālemde sebeb-i necātımız olan 'aleyhi'ş-şalāt ve's-selām Efendimiz Ḥażretlerinig mu'cize-i nübüvvetleri ve hālen serīr-ārā-yı erīke-i şevket cihān-bānī ve revnak-efzā-yı salṭanat saṭvet-i hākānī nāşır-ı ışlāḥ-ı muşāliḥ 'ıbād-ı kāmi '-i ehli'l-bağī ve'l-fesād ḥāfıẓ-ḥūze-i dīn-i mübīn ḥāris-i memālik-i müslimīn el-mü'eyyed bi-te'bīd-i subḥānī ve'l-muvaffak bi-tevfīk-i rabbānī sulṭānu'l berreyn ve hākānu'l baḥreyn hādimü'l-Ḥaremeyni'ş-Serīfeyn illā hüve's-sulṭān ibni's-sulṭān ibni's-sulṭān es-Sulṭān Maḥmūd Hān Ğāzī ibni's-Sulṭān 'Abdu'l-Ḥamīd Hān Ğāzī ibni's-Sulṭān Aḥmed Hān Ğāzī eṭālallāhu 'omrehu ve ebed salṭanatehu ve eyyedallāhu mülkehu ve enfeź hükmehu ḥażretlerinin mahżā kuvvet-i ṭāli ' civān-baht dārā-dirāyet ve te'sīr-i semere-i ma 'deletleri olarak bu 'abd-i biçāre-i 'āciz ve aḥkar ve bende-i nāçiz ve kemter Masdariyecizāde Seyyid Ḥüseyin kulları Mühendishāne-i Ḫümāyūn'da serhalīfelik hidmet-i celīleleriyle müstahdem oldīğīm ecilden otuz seneden beru işbu 'adīmü'l-imkān deyu hendese kitāblarında mestūr olan ve bunca müddetten beru hendese ṭarīķiyle ẓafer-yāb olunmayan muṭlakan bir zāviyenin yāhūd bir kavsın mütesāviyeten üç cüz'e taksīmi huşūşuna ẓafer-yāb olub..."

"With the help of the Almighty God and the miracles of the prophethood of the Holy Prophet (peace and blessings of God be upon him), who is the reason for our salvation in the two worlds, and as a result of the consequences of the justice of Sultan Mahmūd Khan Ghāzī, son of Sultan Abdu'l-Hamīd Khan Ghāzī, son of Sultan Ahmad Khan Ghāzī, who still adorns the great throne, protects the world, enhances the beauty of the sultanate, the irresistible power belonging to the sovereign, and the helper of the reform of the reformers, the one who expelled the rebels and mischief-makers, the protective helmet of the religion, the protector of the kingdom of the Muslims, the Sultan of the two continents and the ruler of the two seas, the servant of Haramayn al-Sharīfayn, backed by the eternalization of Allah (swt), and successful with the support of Allah (swt), this wretched, helpless and despicable servant and insignificant and incomplete slave, I, Maṣdāriyeci Seyyid Ḥūseyin, as the head caliphate at Mühendishāne-i Hūmāyūn, have achieved the problem of dividing an angle or arc into three equal parts, which has been called impossible in geometry books for thirty years and has not been solved by geometric methods for all this time."<sup>6</sup>

The date he gives for his success is Shaban 13th, h.1237. He also mentions that he made his solution confirmed and registered by the lecturers and caliphs of Mühendishāne-i Humāyūn:

<sup>6</sup> Hüseyin Efendi, Teslîs-i Zâviye ve Kavs, 5-6.

"...tārīh-i hicret-i nebeviyyenig işbu bin iki yüz otuz yedi senesi şa 'bān-ı şerīfig on üçüncü günü hendese tarīķi ile mutlaķan bir kavsı mütesāviyeten üç cüz'e taksīm itmek mümkün oldığı yed-i 'ācizānem ile bulunmağın Mühendishāne-i Hümāyūn'un cümle ḥavācesi ve hulefā efendiler kullarına da 'vā-yı meźkūrenig bi'l-burhāni'l-hendesī isbāt olundığı imżā ve temhīr itdirilub..."

"On the thirteenth day of Shaban the year one thousand two hundred and thirty-seven of Hijri calendar, I showed that it was definitively possible to trisect an arc with the help of geometry with my weak power, and all the lecturers and caliph masters of Muhendishāne-i Humāyūn signed and sealed that the mentioned case is proved with the geometry.."<sup>7</sup>

In the printed version of the work, there is indeed a list of the lecturers who approved and signed the solution at the end of the first part. On this signature page we find the following statements:

"Bunca müddetden beru hendese ṭarīķiyle bulunmayub cemī ' ḥükemā ve mühendisīnig müşkili olan te<u>slīs-</u>i zāviye yāhūd te<u>slīs</u>-i ķavs māddesi cemī ' i 'tirāzdan sālim olaraķ hendese ṭarīķiyle ḥall olunub Mühendishāne-i Ḥümāyūn'da cümle muvācehesinde bi'l-burhāni'l-hendese isbāt olunduğunu mübeyyen işbu maḥalle imzā olundu.

Seyyid 'Alī Serhāce-i Mühendishāne-i Hümāyūn Yaḥyā Nācī Hāce-i Sānī-yi Mühendishāne-i Hümāyūn Seyyid Meḥmed Hāce-i Sālīs-i Mühendishāne-i Hümāyūn Seyyid 'Abdu'l-Hālim Hāce-i Rābi '-i Mühendishāne-i Hümāyūn Seyyid 'Alī Halīfe-i Sānī-yi Mühendishāne-i Hümāyūn El-Hāc Seyyid 'Omer Halīfe-i Sālīs-i Mühendishāne-i Hümāyūn Maḥmūd Halīfe-i Rābi '-i Mühendishāne-i Hümāyūn"

"Here put the signs to show that the question of trisecting the angle or the arc, for which a solution had not been found by geometric methods for so long and which had been a difficulty for all scholars and geometricians, was solved by geometric methods, geometrically in the presence of all Muhendishāne-i Humāyūn, free from all objections."

Sayyid 'Alī, the head lecturer of Muhendishāne-i Humāyūn. Yahyā Nācī the second lecturer of Muhendishāne-i Humāyūn Seyyid Mehmed, the third lecturer of Muhendishāne-i Humāyūn Sayyid 'Abdu'l-Ḥālim, the fourth lecturer of Muhendishāne-i Humāyūn Sayyid 'Alī, the second caliph of Muhendishāne-i Humāyūn el-Hāc Sayyid 'Omer, the third caliph of Muhendishāne-i Humāyūn Mahmūd, the fourth caliph of Muhendishāne-i Humāyūn" <sup>8</sup>

<sup>7</sup> Hüseyin Efendi, *Teslîs-i Zâviye ve Kavs*, 6-7.

<sup>8</sup> Hüseyin Efendi, Teslîs-i Zâviye ve Kavs, 22.

Huseyin Efendi also mentions the possible benefits of his solution to other problems that would be solvable by trisecting the angle with Euclidean geometry in the preface. It is undoubtedly a source of pride that such a famous problem, which has not been solved for such a long time, has been proved at the Muhendishāne of the Ottoman Empire. To prevent the solution from falling into the hands of the Europeans and being claimed by them, he concludes the preface with his intention to present it to the Sultan, and his wish that it will be recorded by a chronicler to protect the copyrights and be printed in the official printing house and distributed to all libraries, thus making it available to those interested.

"...zamān-ı medīdeden beru müşkil ve 'adīmü'l-imkān olan mādde bi'l-burhāni'l-hendesī hall olunub mümkün olmağın ve işbu te<u>s</u>lī<u>s</u>-i zāviye māddesi yāhūd te<u>s</u>lī<u>s</u>-i ķavs isti 'ānesiyle bu vaķte ķadar hendese ṭarīķiyle mümkün olmayan mevādd-ı ke<u>s</u>īre bundan şogra mümkün olacağı derkār olmağla fevā'id-i ke<u>s</u>īre ḥāsıl olacağı bedīdār oldığından ol dergāh-ı mülūkāneye hezār 'acz ve ķuşūr ile cür'et-i takdīm kılındı. Egerçi nīm nazar-ı iltifāt-ı tācidār ma 'delet-kāren buyurulur ise ṭāk-bülend-i kāşāne-i iftihār olacağı bī-raīb āşikārdır. ķaldı ki böyle 'ulūm-i garībeden olan mādde-i müşkilig zamān-ı ma 'delet-i şāhānelerinde Devlet-i 'Aliyye-i 'Osmāniyye Mühendishānesi'nde bulunduğı ve Avrupa mühendislerinin şāyed ellerine geçer ise "biz bulduķ" demege ṭarīķ bulmamaķ içun vaķ 'a-nüvīs ma 'rifetiyle tārīhe 'aynen kayd olunması ve yalgız bu mādde tab 'hānede tab ' olunaraķ cemī ' kütübḥānelere vaż ' olunmaklıķ ile 'āleme neşr olunması tensīb buyurulur ise ol vechle tab 'hānede tab ' ile Mühendishāne-i Hümāyūn kütübhānesine ve sā'ir kütübhānelere vaż ' olunması bābında irāde-i kirāmet-i ma 'delet-i şāhāne erzān buyurulması ümīd-i 'ācizāneleridir."

"The problems that seemed difficult and impossible for a long time have been solved using the geometric method. It became obvious that the solution to trisecting angles or arcs problem made it possible to solve many other problems which were previously deemed unsolvable by the geometric method. There are many advantages to this, so it has been presented with a great deal of incapacity and imperfection to the sublime presence of Sultan. It is obvious that it would be a great honor if even half the interest and support of the righteous Sultan is bestoved on it. Moreover, as such an article of knowledge, which is one of the weirdest of all sciences, it was discovered at the Muhendishāne of Devlet-i 'Aliyye-i 'Osmāniyye at the time of the righteous Sultan. It is our weak hope that the will of Sultan's high justice orders it to be recorded in history by a chronicler and printed in the state printing house and sent to Muhendishāne-i Hümāyūn library and other libraries, and released to public in order to prevent European geometricians from them saying "We found it." if they obtained the solution.<sup>9</sup>

<sup>9</sup> Hüseyin Efendi, *Teslîs-i Zâviye ve Kavs*, 7-8.

Sultan Mahmūd II did not refuse Hüseyin Efendi's request and ordered the publication of the work. The document indicating that the work was presented to the Sultan is available at the State Archives.<sup>10</sup> The printed version of the treatise consists of thirty-four pages and two drawings. In the work, two different solutions to the problem are given. Between these two solutions is the approval and signature page of the engineering and mathematics lecturers. There is also a manuscript copy of the work in the ITU Mustafa Inan Library.<sup>11</sup>

The situation up to this point is a common case of an Ottoman scholar publishing the fruits of his efforts to solve a problem in geometry literature in the early nineteenth century. The content of the work and its place in the history of Ottoman mathematics are subjects of a separate study. What we will focus on here is how the discrediting comments about Hüseyin Efendi and his treatise emerged starting from the publication of the work to the present day and whether they are justified or not.

## Why is Masdariyecizāde not considered reliable?

There is not any information on how Masdariyecizāde's work was perceived among mathematicians of his time. About seventy years later, a mathematician and historian of science, Salih Zeki wrote a series of articles on this subject on the *Resimli Gazete*.<sup>12</sup> It is this series of articles that is the basis for today's biased and cynical view of Hüseyin Efendi and the treatise *Teslīs-i Zāviye and Kavs*.

Before going into the details of Salih Zeki's series of articles, it is necessary to mention two recent publications on Maşdariyecizâde Hüseyin Efendi and his treatise on trisecting the angle, on which most of the public opinion are based. The first is the article entitled "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi" (Salih Zeki's Articles on Trisecting the Angle entitled "A Geometry Problem")<sup>13</sup> by A. Bir and M. Kaçar published in *Osmanlı* 

<sup>10</sup> Devlet Arşivleri Başkanlığı Osmanlı Arşivi (OA), Hatt-ı Hümâyûn (HAT) 492/24162.

<sup>11</sup> Hüseyin Efendi, *Teslîs-i Zâviye ve Kavs Risâlesi,* İTÜ Mustafa İnan Kütüphanesi, Nadir Eserler Koleksiyonu, 7081.

Salih Zeki, "Teslīs-i Zāviye Mes'elesi 1-4", *Resimli Gazete*, Cilt1/Yıl 1, 34 (Teşrînievvel 1307/m.1891):
 410-413; 35 (Teşrînisâni 1307/m.1891): 422-426; 36 (Teşrînisâni 1307/m.1891): 434-437; 37 (Teşrînisâni 1307/m.1891): 446-448.

<sup>13</sup> Atilla Bir ve Mustafa Kaçar, "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi", *Osmanlı Bilimi Araştırmaları* VII, no.1 (2005): 45-66.

*Bilimi Araştırmaları* in 2005 and the second is A. Kökcü's article entitled "Resimli Gazete'de 'Teslis-i Zâviye' Meselesi" (The Problem of 'Teslis-i Zâviye' in Resimli Gazete) published in *Dörtöğe* in 2013<sup>14</sup>. The second article is based on the author's master's thesis prepared at Ankara University, Institute of Social Sciences, Department of Islamic History in 2009.<sup>15</sup> The authors of the first article also presented a paper which includes a brief analysis of Masdariyecizāde's treatise, entitled "Osmanlıda Bir Bilim Skandalı: Mühendishâne-i Berri-i Hümâyun Hocası Masdariyecizâde Hüseyin Efendi'nin Teslis-i Zaviye Risâlesi" (A Science Scandal in the Ottoman Empire: Teslīs-i Zāviye Treatise of Muhendishāne-i Berrī-i Hümāyūn Lecturer Masdariyecizāde Hüseyin Efendi) at the 3rd Panel on Science and Engineering Ethics organized in 2011 by the Chamber of Electrical Engineers of the Union of Chambers of Turkish Engineers and Architects.<sup>16</sup>

The first of the problems in Bir and Kaçar's article is already apparent in the title. While the title of the article is "Salih Zeki's Articles on Trisecting the Angle entitled 'A Geometry Problem' ", the original title of Salih Zeki's articles is "The Problem of Trisecting the Angle" and continues for four issues starting from the issue 34. "A Geometry Problem" is the title of another article published before this series of articles, in the issue 29, which consists of a letter from an anonymous person raising the question of trisecting the angle and demanding a solution.<sup>17</sup> The authors have confused the titles of the two articles.

In the introduction of Bir and Kaçar's article, it is stated that the study is about Salih Zeki's "opinions and thoughts, and the general solution and the proof of geometric unsolvability of the problem" regarding the problem sent to the newspaper. From this expression, we understand that Bir and Kaçar's article is to be a study on Salih Zeki's article. However, when we read the article in parallel with Salih Zeki's series of articles, we see that it is not about him but rather an expression of him in contemporary Turkish. As can be seen in the excerpts we will examine below, Salih Zeki's article is quoted word for word, from the footnotes to

<sup>14</sup> Ayşe Kökcü, "Resimli Gazete'de "Teslis-i Zâviye Meselesi", Dörtöğe 2, no.4 (Ekim 2013): 121-138.

<sup>15</sup> Ayşe Kökcü, *Resimli Gazete*'de Teslis-i Zâviye Meselesi, (M.A. Thesis, Ankara Üniversitesi, 2009, unpublished).

<sup>16</sup> Atilla Bir ve Mustafa Kaçar, "Osmanlıda Bir Bilim Skandalı: Mühendishâne-i Berri-i Hümâyun Hocası Masdariyecizâde Hüseyin Efendi'nin Teslis-i Zaviye Risâlesi", TMMOB Elektrik Mühendisleri Odası 3. Bilim ve Mühendislik Etiği Paneli, (Nisan 2011).

<sup>17</sup> Anonymous, "Bir Hendese Meselesi", Resimli Gazete, Cilt 1/Yıl 1, 29, (Teşrînievvel 1307/m.1891): 360.

the adverbs used at the beginning of the sentences. Leaving it to the discretion of the reader and other researchers as to what class such a work should be among academic writings or if their article merely a linguistic simplification of the original work, whether it is an ethical obligation to state it clearly, let us examine the other article.

The second article by Ayşe Kökcü promises to investigate the first series of articles written by Salih Zeki Bey in *Resimli Gazete* about Hüseyin Efendi's solution and the history of the problem, and the criticisms he made of the solution found by an İbrahim Efendi published a few issues later. However, most of the article deals with the history of the problem of trisecting the angle from ancient times until this time. The articles in *Resimli Gazete* are summarized in a few pages at the end. The section on Masdariyecizāde, which is of interest to us, is a summary of the article by Bir and Kaçar mentioned above and consists of about one page. The most significant evidence of Kökcü's reference to the Bir and Kaçar's article is that she repeats the mistake regarding the name of Salih Zeki's series of articles. Let us defer this issue to our analysis of Bir and Kaçar's article and note a few sentences in Kökcü's article that can be good evidence of a problematic scientific historiography mentality regarding Masdariyecizāde's solution:

"The solution method described by Masdariyecizāde in the second part of his treatise is most probably that of François Viète, the greatest French mathematician of the sixteenth century, based on the solution proposed by one of the Greek mathematicians, Archimedes (3 B.C.E); for how the question is treated *resembles* that of Viète, and *it is possible* to learn this method from the relevant section of the Encyclopédie Méthodique."<sup>18</sup>

Since Kökcü could not even attribute this incomplete solution to Masdariyecizāde, she fell prey to the weakness that some researchers display especially regarding the works of scholars of the eighteenth and nineteenth century on modern sciences, and pursued the question: "From whom he got it?" To prove this prejudice, she immediately follows with a judgement full of contradictions:

"Masdariyecizāde has not stated *whom* this traditional method *belongs to* and *where he got it from*, probably because it was a well-known and well-recognized method among the lecturers of Muhendishāne, and instead he has simply presented it as another proof."<sup>19</sup>

<sup>18</sup> Kökcü, "Resimli Gazete'de "Teslis-i Zâviye Meselesi", 130. [Italics are ours.]

<sup>19</sup> Kökcü, "Resimli Gazete'de "Teslis-i Zâviye Meselesi", 130. [Italics are ours.]

Kökcü also mentions in her article that Hüseyin Efendi had his work confirmed by the lecturers of Muhendishāne. In this case, how is it possible that Hüseyin Efendi appropriated a solution known to everyone and secure the fame and financial benefits of this discovery while his colleagues acknowledge the solution as an invention? We believe the judgment of taking the solution from Western soruces is the author's personal interpretation and cannot be considered as evidence.

In her article, Kökcü mentions the treatise of Hüseyin Efendi to this extent and moves on to another person's solution on the topic published in *Resimli Gazete*. Although it is pretty problematic in terms of academic perspective and method of scientific historiography, we content ourselves with that much about this article, since its content, scope, and impact are narrower compared to Bir and Kaçar's article mentioned above.

It would be appropriate to read the comments in Bir and Kaçar's article about trisecting the angle and Huseyin Efendi (quoted by Salih Zeki) from Salih Zeki's own original words, and not from second hand. Salih Zeki begins the first one of a series of four articles entitled "The Problem of Trisecting the Angle" by explaining why he wrote such an article. He mentions the article in the 29th issue of the newspaper titled "Bir Hendese Mes'elesi", which brought up the problem of trisecting the angle, and says that this problem has not been solved for two thousand years; however it has been known for two hundred and fifty years that its solution is not possible through geometry.

"Ğazetemizin yirmi doķuzuncu nüshasına bir zāt ṭarafından "Hendese Mes'elesi" serlevḥası altında ḥalli maṭlūb bir mes'ele derç itdirilmiş ve nüsha-i meźkūreyi müṭāla 'a buyuranlarca ma 'lūm oldığı üzere mes'elede teslīs-i zāviye māddesinden ya 'nī doķsan dereceden dūn olan bir zāviyeyi hendese-i 'ādiyye ṭarīkiyle üç müsāvī kısma taksīm itmekden 'ibāret bulunmuş idi.

İki bin bu kadar seneden beri hendese-i 'ādiyye ṭarīkiyle ḥall olunamayan ve iki yüz elli seneden beri de o şūretle ḥallinin 'adem-i imkānına beyne'l- 'ulemā ḥükm olunan bir mes'eleyi ara şıra tāzeleşdirmek ve hendese-i 'ādiyye ṭarīkiyle ḥalline çalışmak mes'elenin esāsen neden 'ibāret oldığına ve ne için ḥall olunamadığına vākıf olmamakdan ileri geleceği ve çünkü mes'ele-i meźkūrenig hendese ṭarīkiyle ḥalline imkān olmadığı ḥakkında berāhin-i riyāżiyye mevcūd oldığını bilen her şāḥib-i 'aklın bu gibi beyhūde taḥarriyātdan şarf-ı nazar ideceği şüpheden vāreste bulunmuşdur."

"In the twenty-ninth issue of our newspaper, a problem was published under the title of "Hendese Mes'elesi" by a certain person and, as it is known to those who have read the issue, the problem consisted of trisecting the angle, that is, to divide an angle of less than ninety degrees into three equal parts using conventional geometry.

To occasionally refresh a problem that has not been solved by conventional geometry for about two thousand years, and has been discussed among scholars its impossibility to be solved in this way, and to try to solve it still by conventional geometry, is due to the lack of understanding of what the problem consists of and why it cannot be solved. It is beyond doubt that any rational person who knows that there is mathematical proof of the impossibility of solving the mentioned problem through geometry, will avoid such futile investigations."<sup>20</sup>

He explains that it is futile to deal with this unsolvable problem. However, people occaisonally come forward claiming a solution, and some even approached him on the subject. To prevent these attempts, he decided to study the problem in depth. He intended to write this series of articles to explain the problem mathematically, talk about its history and show why it cannot be solved, prove that solving the problem with geometry is absurd.

"İşte şu nokţa-i mühimmeye vākıf olmayarak el-yevm mes'elenin bi'l-hendese halli mümkündür iddi 'āsında bulunan kimseler görüldigi ve hattā mūmā ileyhimin 'adedi günden güne tezāyüd itmekde oldığı maţba 'aya gönderilen evrāk ile müsbet bulunduğı cihetle şu iddi 'ānıg ne kadar bātıl oldığınu ve mes'ele-i meźkūrenin bi'l-hendese halline çalışmak 'ādetā 'abesle iştigāl dimek oldığınu isbāt ve beyān zımnında teslīs-i zāviye mes'elesinig riyāziyye nokţa-i nazarınca neden 'ibāret oldığıyla mücmelen tārīhine ve 'alā'l-huşūş bi'l-hendese ne için hall olunamadığına dā'ir ber vech-i ātī bir makālenin neşri münāsib görülmüşdür."

Since there are people who claim that the problem can be solved with geometry without knowing this vital point, and since it is evident from the documents sent to the printing house that the number of those people is increasing day by day, to emphasize how false this claim is and it is almost absurd to try to solve the said problem with geometry, it has been deemed appropriate to publish an article on what the problem of trisecting the angle consists of according to the mathematical point of view, its brief history and, above all, on why it could not be solved through geometry.<sup>21</sup>

After this introduction, Salih Zeki explains the problem geometrically and then gives a solution under the subtitle "What the problem consists of". However, since this solution does not meet the desired conditions, i.e., it contains elements other than line and circle, it is not solved with a straightedge and a compass. Salih Zeki himself states that this is not the desired solution.

<sup>20</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 1", 410.

<sup>21</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 1", 411.

Then follows the heading "The concise history of the problem". This section is quite long and detailed.

In the chapter that begins with the words

"Müteķaddimīn teslīs\_i zāviye ve tad ʿīf-i muķa ʿab mes'elesini ḥall içun yalgızca ķuṭū ʿ-i maḥrūtiyyāti isti ʿmāl ile iktifā itmemişler, bu yolda şarf-ı mesā ʿī iderek ḥaķīķaten ʿulūm-i riyāżiyyece mühim keşfiyāt vücūda getirmişlerdir."

The ancients were not contended merely using conic sections to solve the problem of trisecting the angle and doubling the cube, and they made important discoveries in mathematical sciences by working hard in this way.<sup>22</sup>

Salih Zeki gives an overview of the history of the problem from the Hellenistic period onwards. He mentions methods and tools used by people interested in the subject in different eras and civilizations. Almost all of these solutions use curves consisting of conic sections. After briefly mentioning the conchoid curves used by Nicomedes and the cissoid curves of Diocles from the Hellenistic period, he continues by saying:

"Mütekaddimīnig zihinlerini bu ķadar işgāl iden şu mādde şonraları müte'ahhirīnig de naẓar-ı diķķatlerini celbe başlamış ve hendese-i 'ādiyye ile ḥalli mümkün olamaması mūmā ileyhimi dürlü dürlü keşfiyāt ve taḥarriyāta sevk iylemişdir."

This problem, which so occupied the minds of the ancients, later attracted the attention of moderns, and the impossibility of solving it with conventional geometry led them to various discoveries and research.<sup>23</sup>

He notes that some of the scholars who thought that the problem could not be solved with geometry in the desired way preferred to incorporate other drawing methods to obtain conchoidal, cissoidal, or conic curves to use in the solution, while others followed the path of discovering new curves. He mentions some European scholars such as François Viète, Vincenzo Viviani, and Isaac Barrow.

After that, it is time to explain his interpretations, which constitute the key point of our research. Salih Zeki believes that it has long been mathematically proven in Europe that the problem cannot be solved with a straightedge and a

<sup>22</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 422.

<sup>23</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 422-423.

compass as desired. Nevertheless, he is astonished that there are some who claim to have found a solution. As mentioned at the beginning, one of the reason for starting this series of articles is the hope that these attempts will now end.

"Asıl ğarībi şurasıdır ki müte'ahhirīnden ba 'zılarınıg mes'elenig hendese-i 'ādiyye i 'ānesiyle ya 'nī cedvel taḥtası ve per-kār vāsıṭasıyla ḥall olunamayacağı 'ulūm-i riyāziyye ile kat 'iyyen isbāt olundukdan sonra dahi yine bu yolda it 'āb-ı fikr itmekden geri turmamışlardır. Bu gibilerine yakın vakte kadar Avrupa'nın ekser medārisinde teşādüf olundığı gibi şimdileri böyle ötede beride ara şıra zuhūr itmekde bulunmuşlardır."

"The strangest thing is that some of the later scholars did not hesitate to make an effort to understand the subject in this way, even after the science of mathematics had proved that the problem could not be solved with the help of conventional geometry, i.e. with the help of a straightedge and a compass. Until recently such people could be found in most of European schools, and they have been occasionally appearing here and there from time to time."<sup>24</sup>

He mentions Masdariyecizāde Hüseyin Efendi as an outstanding example of those who occasionally come up with the claim of a solution in the Ottoman Empire.

"İşte bu kabilden olarak bin iki yüz otuz yedi sene-i hicriyyesinde Mühendishāne-i Berrī-yi Hümāyūn'da mu 'allim bulunan Maşdāriyecizāde Hüseyin Efendi nāmında bir źātıg da bu mes'eleyi hall itmek sevdāsına düşdügü görülmüşdür. Mua 'llim-i mūmā ileyh teşlīş-i zāviyeyi hendese-i 'ādiyye țarīkiyle hall itdim iddi 'āsıyla keyfiyyetin vak 'anüvīs ma 'rifetiyle żabț ve tārīhe kaydını cānib-i hükūmet-i seniyyeden istid 'ā itmiş ve isbât-ı müddi 'â żımnında birkaç źâta da şehādet itdirmişdir ki ona dā'ir olan maţbū ' risālesi şāhidlerig mühürleriyle mahtūm oldığı hālde hemān ek<u>s</u>er kütübhānelerde mevcūd bulunmakdadır."

Similarly, it was seen that a person named Masdariyecizāde Hüseyin Efendi, who was a lecturer at Muhendishāne-i Berrī-yi Humāyūn in the Hijri year one thousand two hundred and thirty-seven, *fell in love* with solving the problem. The said lecturer demanded by the supreme government that the situation be recorded by a chronicler in history, claiming that he had solved the problem of trisecting the angle by conventional geometry, and *he made several persons testify to prove the claim*. His article, sealed by witnesses, is available in many libraries.<sup>25</sup>

<sup>24</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 423.

<sup>25</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 423. [Italics are ours.]

Salih Zeki's tone here will influence the sarcastic attitude in later literature. He describes Masdariyecizāde's effort to prove as "falling in love with solving the problem." Obviously, he thinks that Hüseyin Efendi was engaged in a futile endeavor whose end is evident from the beginning. Moreover, he had recorded this futile effort by "referring to the testimony of several people to prove his claim."

Few people today read the story from Salih Zeki's article. In Bir and Kaçar's article, which readers usually consult on the subject, the relevant part is nothing more than adding a few words to the intralingual translation of Salih Zeki's interpretations, which would make Hüseyin Efendi even more ridiculous in this desperate effort:

"For example, Masdariyecizāde Hüseyin Efendi, one of the lecturers at Muhendishāne-i Berrī-i Hümāyūn, also *fell in love with solving this problem* on Shaban 13, 1237 (May 5, 1822). Claiming "I trisected an angle", Hüseyin Efendi recorded the situation through a historian and obtained permission from the relevant authorities *to inscribe his name in the history of science*. He wanted to support the proof of his claim by listing some of the lecturers of Muhendishāne as witnesses."<sup>26</sup>

The authors repeated their comments in their panel paper on the subject. This is the information that is referred to today when it comes to Maşdāriyecizāde Hüseyin Efendi and his treatise *Teslīs-i Zāviye and Ķavs*. Recalling that the primary source of this information is Salih Zeki's comments, let us return to investigating his rightness.

# The Interpretations of Salih Zeki on Teslīs-i Zāviye ve Masdariyecizāde

After referring to Hüseyin Efendi and his work, Salih Zeki states that such solution attempts only yields approximate solutions, and that although positive results are obtained with the methods that solves the problem with curves other than circle, it is only after the discovery of analytical geometry that the impossibility of the desired solution by using only a straightedge and a compass became clear. He thus relates the subject to René Descartes (d. 1650). For he claims that the mathematical proof of the impossibility was given in Descartes' work *La Geometrié* two hundred and fifty years ago before his time:

<sup>26</sup> Bir ve Kaçar, "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi", 52. [The italics are ours.]

"Mütekaddimīn cebrig hendeseye tatbīķini bilmedikleri cihetle bir zāviyenig hendese-i 'ādiyye ile üç müsāvī ķısma taksīm idilmesinin 'adem-i imkānına da bir sebeb-i ma 'kūl bulamıyorlar ve ma 'a mā-fīh ḥall olunamadığını görerek müteḥayyir kalıyorlar idi.

Vaķtā ki meşhūr Deķart [Descartes] bundan iki yüz elli sene evveline gelinceye ķadar riyāziyyātıg ayrı ayrı nazar-ı muţāla 'aya alınan iki büyük şu 'besini tevhīd iderek "hendese-i halliye" nāmı taḥtında bir fenn-i müstaķil teşkīl itdi, teslīs\_i zāviye mes'elesinin de niçin bi'l-hendese ḥall olunamadığı derḥāl ma 'lūm ve āşikār oldu."

As ancients did not know the application of algebra to geometry, they could not find any rational reason for the impossibility of trisecting the angle with conventional geometry. They remained puzzled by seeing that it could not be done.

When the famous Descartes created a science of its own called "analytic geometry" by uniting the two major branches of mathematics that had been studied separately two hundred and fifty years ago, it immediately became known and obvious why the problem of trisecting the angle could not be solved with geometry.<sup>27</sup>

Salih Zeki analyzed Descartes' key place on the subject in the rest of the series. Before that, in the subsection entitled "Why the problem cannot be solved with geometry?" he attempts a mathematical explanation of why the problem cannot be solved in its original form, using only conventional geometry. This section is quite detailed. Indeed, it starts in the second article of the series, continues in the third one, and occupies part of the fourth.

There are severe problems regarding this section in the article of Bir and Kaçar. Although the authors do not state that they did a word-for-word translation of Salih Zeki, if the two texts are read side by side, it will be realized that their text is a translation. Nevertheless, it is evident that they have not been precise enough with the mathematical terms, inferences, and interpretations, and they conveyed much information incorrectly. This is an example of the necessity of help of an expert when studying a scientific text. In this case, the complete transliteration of Salih Zeki's text is still unavailable, and the text of these authors is the only source on this subject in contemporary Turkish. Nevertheless, a scientifically inaccurate translation serves neither to understand Salih Zeki nor the subject he discussed.

The errors are so grave that even reading the two texts without comparison, it is evident that there is technical confusion. Since our study does not extend to the mathematical content of Salih Zeki's article, we limit ourselves to correcting some of these errors. However, these examples show that these important scientific texts of Salih Zeki require a second and more precise study.

In the section explaining the unsolvability of the problem, Salih Zeki proceeds from two conclusions. The first is the theorem known as the "Fundamental Theorem of Algebra", which states that a polynomial of degree n must have n real or complex roots. Although the theorem has been on the agenda of European mathematicians since the seventeenth century and various attempts have been made to prove it, the first consistent proof was given by Jean-Robert Argand (d. 1822) in 1806, followed by Carl Friedrich Gauss (d. 1855) who gave an algebraic proof in 1816 in place of the incomplete geometric proof he had done earlier.

Salih Zeki mentions this theorem and then states that some of the roots may be complex. However, if they are real, they will not harm the absolute results of the general theory.

"Mevādd-1 mezkūreden birincisi ya 'nī mu 'ādelāt-1 cebriyyenig naẓariyāt1 mūcibince bir mu 'ādele-i cebriyye her ķaçınc1 dereceden ise o mu 'ādelenig o ķadar 'adedde cezri ya 'nī mechūlünüg ķıymeti olmaķ lāzım gelir. Gerçi şu cezrlerden ba 'zıs1 muḥdes olarak ẓuhūr ider ise de şu ḥāl-i meźkūr cezrlerig ḥakīkī olduklar1 takdīrde naẓariyye-i 'umūmiyyeden istihrāc olunacak netāyic-i muṭlakaya halel ibrās itmez."

Under the first of the matters mentioned above, namely the theory of algebraic equations, every algebraic equation must have the same number of roots with its degree, namely the value of the unknowns of the equation. Although some of these roots could be complex, this does not influence the absolute conclusions which can be drawn from the general theory, if the roots are real.<sup>28</sup>

### Bir and Kaçar cites this part as follows:

"The first of these points, namely the fundamental theorem of algebra, is the principle that whatever the order of an algebraic equation, it must have the same number of roots, i.e., its unknown value. Although some of these roots may be imaginary, the conclusion derived from the general theory may not always correspond to a solution, even if the roots are real."<sup>29</sup>

<sup>28</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 424.

<sup>29</sup> Bir ve Kaçar, "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi", 55.

As can be seen, this statement is neither translationally nor mathematically correct. In the second sentence, it was said that some of the roots could be complex, and then a meaningless sentence is formed as real roots do not always give a solution.

The second mathematical conclusion that Salih Zeki mentions is the role of the number and character of the roots in drawing the curves for the solution. Accordingly, geometric representation of the roots of an algebraic equation is achieved by drawing two lines (straight lines or curves) that intersect each other in as many points as the number of roots. That is, the number of roots determines the number of intersection points. However, while real roots are represented in the analytical plane, it is not possible to represent complex roots. In this case, only a geometric representation of real roots is possible.

This discussion takes a completely different turn in the translation of Bir and Kaçar. Since the authors confused the terms real and complex (*muḥdes*) roots with positive and negative roots, they use the word "positive" instead of "real" and "negative" instead of "complex" beginning from this part of the article onwards. This serious mistake leads incomprehensible sentences that are mathematically incorrect.

This paragraph of Salih Zeki

"Huşūṣāt-1 meşrūḥa cezrleri kāmilen ḥaķīķī olan bir muʿādele için pek ẓāhir bir şey ise de muʿādele-i meźkūrenig cezrlerinden baʿżısı muḥdes oldığı ḥālde ta 'yīn olunacak kıymetlerig 'adedi, muʿādelenig derecesinden dūn bulunacağından naẓariyyenig kat 'iyyetine halel gelir ẓann olunur ve cezrler meyānında muḥdes bulunmadığı şūretde dahi muʿādeleyi tersīme hidmet idecek olan kemmiyyet-i vaż 'iyyelerig muʿādelenig derecesi 'adedince nukāṭda yekdigerini kat ' itmesine lüzūm olmadığına źāhib olunur."

Although explanations are very clear for an equation whose roots are completely real, if some of the roots are complex, the accuracy of the theory is considered damaged, since the number of values to be determined is less than the degree of the equation, and even in the absence of any complex root subsequently, it is not necessary for the quantities that will be used to draw the equation to intersect each other at as many points as the degree of the equation.<sup>30</sup>

#### has become such in Bir and Kaçar:

<sup>30</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 424.

"The special case described above applies to an equation of which all the roots are positive; however, if some roots in the equation are negative, it can be assumed that the accuracy of the theory is lost because the number of values to be determined is less than the degree of the equation. If there are negative roots among the roots, the intersections that will be used to draw the equation may lead to the conclusion that it is not necessary to intersect at points as much as the degree of the curve equation, but this assumption and thought is based on being deceived by appearances."<sup>31</sup>

Apart from the wrong use of "positive" instead of "real", "negative" instead of "imaginary", while Salih Zeki says "in the absence of imaginary roots", the text of Bir and Kaçar says "in the presence of negative roots". In that point, the mathematical consistency is completely lost.

This confusion continues in the next paragraph. While Salih Zeki says that if one or more roots of an equation are complex, these roots cannot have a geometric meaning:

"Fi'l-ḥaķīķa bir mu'ādele-i cebriyyenig hāvī oldığı mechūlün cezrlerinden biri veya birkaçı muḥdes oldığı şūretde hendese-i ḥalliyece muḥdes kemmiyāta bir ma 'nā-yı hendesī verilemediğinden bi't-ṭabi ' kıyem-i meźkūreye 'â'id bulunan nokṭalar da şeklen irā'e idilemez."

In cases where one or more roots of the unknown in an algebraic equation are complex, the points that belong to the mentioned values cannot be represented geometrically as complex quantities in analytical geometry have no geometrical meaning, naturally.<sup>32</sup>

In Bir and Kaçar, this problem leads to a fundamental error that negative numbers cannot be represented on the analytic plane:

"Indeed, if one or more of the unknown roots of an algebraic equation are negative, the points associated with these numbers cannot, of course, be formally represented, since negative numbers have no geometrical meaning in analytical geometry."<sup>33</sup>

A terminology error is a mistake, but one may considered it as not a very serious one. While such an error might be excused in case of an extreme term in a specialized subject, considering that analytical geometry is already widespread starting from high schools, even someone who has not seen Salih Zeki's own text will recognize that the term "negative" is inconsistent with the text. This crucial

<sup>31</sup> Bir ve Kaçar, "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi", 55.

<sup>32</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 424.

<sup>33</sup> Bir ve Kaçar, "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi", 55.

point that was missed by the authors was apparently not noticed by the referees of the journal either. This is an important example of how research on the history of mathematics can result in a disastrous outcome, if authors and referees as well as editors don not pay attention to even the primary concepts of a fundamental subject such as analytical geometry.

After these two technical details, Salih Zeki states that there are recent studies on geometric representation such as the quaternions of William R. Hamilton (d. 1865), whom he recognizes as a famous mathematician. Also, he mentions to his contemporary Vidinli Tevfik Pasha (d.1901) and his book entitled *Linear Algebra*. Here, too, we come across the test of Bir and Kaçar with the word "*muḥdes*" (imaginary). Here is Salih Zeki's paragraph:

"Ḥattā İngiltere 'ulemāsından müteveffā meşhūr Ḥamilton'un "kuaterniyon" tesmiye itdiği uşūl-i ḥisāb ile kudret-i 'ilmiyyesi müsellem-i enām olan meşāhir-i riyâżiyyūndan ve ferīkān-ı kirāmdan Nāfi 'a ve Ticāret Nāzır-ı 'Ālīsi sa 'adetlü Tevfīk Paşa Ḥażretlerinig ihtirā '-kerdeleri olub İngilizce te'līf iyledikleri bir kitābda mestūr olan cebr-i haṭṭīnig kavā 'id-i esāsiyyesinden biri de kemmiyāt-ı muḥdesenig şeklen irā'esi māddesinden 'ibāret bulunmuşdur."

"One of the aims of the method of calculation called "quaternion" by the late Hamilton, one of the famous scholars of England, and linear algebra that can be found in the book by Tevfik Pacha, Minister of Finance and Trade, one of the commanders of the army and famous mathematician whose scientific skills is recognized by public, is to draw imaginary quantities as geometric forms."<sup>34</sup>

While Salih Zeki notes that one of the basic rules of quaternions and linear algebra is "to draw complex numbers"," in Bir and Kaçar, these words turns into a method of calculation called quaternion and the sentence

"According to a basic rule developed in the book Linear Algebra, it is discussed how to express obtained solutions as geometric forms."<sup>35</sup>

It is now widely known that both Hamilton and Tevfik Pasha aimed to extend algebra of complex numbers. However, Bir and Kaçar transformed the phrase "complex quantities" in the original text to "solution values obtained".

<sup>34</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 2", 425.

<sup>35</sup> Bir ve Kaçar, "Salih Zeki'nin 'Teslis-i Zaviye' Konusundaki "Bir Hendese Meselesi" Adlı Yazı Dizisi", 56.

Before leaving aside the task of comparing the rest of the article with the original, we would like to explain why we are addressing this issue. The negative comments on Masdariyecizāde Ḥüseyin Efendi's work in relation to our subject are based first on Bir and Kaçar's article, and then on Salih Zeki. While the authority attributed to Salih Zeki on the subject is open to question, the reliability of an article that does not even quoted him without error is questionable. This is because the authors did not confirm even the simplest mathematical topics. And yet, the question arises; to what extent will second-hand comments do justice Hüseyin Efendi?

After explaining the two important results mentioned above, Salih Zeki begins his mathematical proof on why the problem cannot be solved using a straightedge and a compass. Since the proof is too extensive so that it should be subject of another study, we content ourselves with few sentences summarizing the argument: In his explanation, Salih Zeki transforms the problem into a geometrical restatement of the roots of a cubic equation. In accordance with the algebraic rules mentioned above, he states that in order to represent three real roots of a cubic equation geometrically, one should draw lines that intersect each other at three points. However, any two circles and lines can have only two intersections. Therefore, it is not possible to represent all the roots of the equation by drawing only circles and lines.

Salih Zeki bestows Descartes the credit of being the first to prove impossibility of solving the problem of trisecting the angle with a straightedge and a compass. Accordingly, the line that divides an angle or arc into three equal parts must be a curve of at least third degree. If this curve is to be obtained by means of two other curves, one must be a circle and the other a quadratic curve other than a circle.

"Teslīs-i zāviye mes'elesinig bi'l-hendese hall olunamayacağını en evvel isbāt eden hendese-i halliyenig mūcidi meşhūr Dekart'dır [Descartes]. 'Ālim-i mūmā ileyh bir kavsı üç müsāvī kısma taksīm idecek hat yalgız bir münhanīden 'ibāret oldığı hālde lā ekall üçüncü dereceden olması lāzım geleceğini ve iki münhanīnig terekkübünden hā~ıl oldığı ~ūretde bunlardan birinig muhīt-i dā'ire, digerinig dā'ireden ğayrı bir ikinci derece münhanīsinden 'ibāret olacağını beyān itmiş…"

It was the famous Descartes, the inventor of analytical geometry, who first proved that the problem of trisecting the angle cannot be solved by geometry. The scholar declared that the line trisecting the arc must be at least of third-degree if it consists of only one curve, and that in the case of a combination of two curves, one of them must be a circle and the other one a quadratic curve other than the circle.<sup>36</sup>

<sup>36</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 4", 448.

The fourth and last article of the series ends with the conclusion that, although there is clear proof that the problem of trisecting the angle cannot be solved with a straightedge and a compass, the attempt to make it possible or even to believe that it is possible is due to ignorance of mathematics and lack of understanding of the basics of mathematics.

"İşte teslīs-izāviye mes'elesinig 'ādī per-kār ve cedvel taḥtası ile ḥalli ğayr-i mümkün oldığına dā'ir elde bir burhān-ı ķaviyy var iken bunu sāḥa-i imkāna sokmağa çalışmak degil ḥatta dā'ire-i imkānda oldığını taşavvur itmek, riyāżiyyeden makṣad ne oldığını bilememekden ve mukaddimāt-ı riyāżiyyeye vukūfu olmamakdan ileri gelir bir keyfiyyetdir."

While there is strong proof that it is impossible to solve the problem of trisecting the angle with a straightedge and a compass, let alone trying to bring it within the realm of possibility, imagining that it is within the realm of possibility is a status that results from not knowing what the purpose of mathematics is and from not knowing the introduction to mathematics.<sup>37</sup>

After condemning those who dream of solving the problem that has been unsolved for two thousand years, and so making history, Salih Zeki adds the warning that it would cause shame to claim to solve it when there is proof of unsolvability.

"Vāķī 'a iki bin bu ķadar seneden beri bunca e 'āẓimin ḥall idemedikleri mes'eleyi ḥall iderek tārīh-i 'ulūmda ibķā-yī nāma çalışmak arzu olunacak bir şey ise de ḥall olamayacağī ḥakkında meydānda böyle bir burhān-ī ķāṭī ' mevcūd iken ḥall iderim iddi 'āsīna ķalkışmanīg bi'l-āhare mūcib-i hīcālet olacağını düşünmek de lāzīmdīr."

Indeed, it is desirable to try to make a name for oneself in the history of sciences by settling an issue that so many scholars have not been able to solve for so many years, yet it is also necessary to think that attempting to claim that one can settle an issue when there is such conclusive evidence that it cannot be solved will bring disgrace in the future.<sup>38</sup>

This is the story of the case we are trying to present as briefly as possible. When Salih Zeki saw those who are still dealing with the problem of trisecting the angle in his time, he condemned those who still did not know the result, with the judgment that Descartes solved the problem two hundred and fifty years

<sup>37</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 4", 448.

<sup>38</sup> Salih Zeki, "Teslīs-i Zāviye Mes'elesi 4", 448.

ago. He even retrospectively criticizes a case that took place about seventy years before him. Contemporary literature on the subject, however, is content to quote and reproduce Salih Zeki's remarks from a hundred years ago, and even makes a mountain out of a molehill and calls Maṣdāriyecizāde's work a "scandal."<sup>39</sup> Thus, Hüseyin Efendi did the rounds as an example of the Ottoman intellectual who was backward compared to Europe and engaged in idle pursuits.

At this point, the question arises: "Is Salih Zeki an authority on the history of science whose word can be taken as law without question?" We will now seek the answer to this question.

## Where was the mistake?

In the series of articles, Salih Zeki states that the first proof of the impossibility of trisecting the angle was given by Descartes. Although he gives neither a citation nor a date, the work in question is the relevant chapter in *La Geometrié*. In fact, this work was translated into Turkish by Yahyazāde Mehmed Rūhiddīn Efendi (d. 1847). Although Salih Zeki does not mention this, Bir and Kaçar accuse Hüseyin Efendi of not being aware of this work. However, we have no evidence for this. We think it more likely that Hüseyin Efendi knew this important translation by Rūhiddīn Efendi, who was in the same circles as him, than not. Even if he was not aware of it, it is not plausible that none of the lecturers at Muhendishāne who approved his work knew about it.

In this case, the question arises, "If he was aware of it, why did he claim to have solved an impossible problem?" In fact, this question has a hidden presupposition: the accuracy of the data presented by Salih Zeki. We know that the declaration of Paris Academy of Sciences in 1775 that it would not accept applications for solving the problems of trisecting the angle and doubling the cube was largely based on Descartes' work. In addition to Descartes' worldwide fame, Salih Zeki's citation of the proof of impossibility with reference to Descartes is probably due to this declaration.

The contemporary literature on the history of mathematics, on the other hand, is not as certain as Salih Zeki when it comes to impossibility proofs. Today, the consensus is that Pierre Wantzel (d. 1848) provided these proofs in 1837.

<sup>39</sup> Bir ve Kaçar, "Osmanlıda Bir Bilim Skandalı".

Although Descartes rightly enjoys the reputation of having been one of the first to lead and strive for impossibility proofs, his arguments do not provide a conclusive one. According to J. Lützen, who elaborated on the proofs of Descartes and Wantzel, Descartes reduced both problems to equations and claimed that the roots of these equations are values that cannot be drawn with a straightedge and a compass. The argument with which Descartes tries to prove this claim gives the impression that he aims to provide an algebraic proof. But in fact, he did not claim to give such proof. Indeed, it turns out that his conclusions are geometrical. When we examine this discussion, we see that it is far from being a coherent reasoning.<sup>40</sup> As a matter of fact, Descartes' attempted proof did not put an end to the problem, and the search for a consistent proof continued for the next two centuries.

Let us read the argument Salih Zeki refers to Descartes in Descartes' own words:

"Inasmuch as the curvature of a circle depends upon a simple relation between the center and all points on the circumference, the circle can only be used to determine a single point between two extremes, as, for example, to find one mean proportional between two given lines or to bisect a given arc; while, on the other hand, since the curvature of the conic sections always depends upon two different things, it can be used to determine two different points." <sup>41</sup>

Another claim of Descartes precedes this argument: A geometric problem that can be solved with a straightedge and a compass corresponds to a quadratic equation. The roots of the quadratic equation obtained by accepting this claim can also be constructed with a straightedge and a compass. Lützen says that Descartes failed to prove this claim, but rather tried to show the opposite, namely that the roots of a quadratic equation can be drawn with a straightedge and a compass. According to Lützen, one reason why such logical errors are common in *La Geometrié* is the rhetorical strategies designed by the author to convince the reader of the power of his method. Here, too, despite the rhetoric, Descartes failed to prove his assertion.<sup>42</sup>

<sup>40</sup> Jesper Lützen, "The Algebra of Geometric Impossibility: Descartes and Montucla on the Impossibility of the Duplication of the Cube and the Trisection of the Angle", *Centaurus* 52 (2010): 4-37, 12.

<sup>41</sup> René Descartes, La Geometrie, (1657), David Richeson, Tales of Impossibility: The 2000-Year Quest to Solve the Mathematical Problems of Antiquity (Princeton: Princeton University Press, 2019): 259.

<sup>42</sup> Jesper Lützen, "The Algebra of Geometric Impossibility", 14.

In his book *Tales of Impossibility*, in which he examines the history of impossibility proofs in mathematics, D. Richeson notes that despite this unsuccessful proof attempt, Descartes made significant contributions to the proofs of impossibility for the problems of straightedge and compass. He revealed that the argument of impossibility was not a vague claim that could not be solved, but a provable theorem, and he transformed geometric problems to algebraic ones and invented algebraic methods that would later be used to solve the problem.<sup>43</sup>

After Descartes' proof attempt, Jean-Étienne Montucla (d. 1799) and Gauss also made algebraic contributions to the proofs of the impossibility of solving these ancient problems with a straightedge and a compass. Not all of them focus on trisection problem, but solutions methods were similar. Therefore, similar methods can be applied to proofs of impossibility. For example, Gauss dealt with the problem of drawing a regular heptadecagon (i.e., 17-gon). In 1801, in his work Disquisitiones Arithmeticae, he gave a rule for drawing all regular polygons, but he did not include the proof due to the lack of space in the paper, a usual excuse at that time. Nevertheless, due to his reputation and authority in the mathematical community, history books generally give his name as the one who solved the problem first. On the other hand, Wantzel, a 23-year-old young mathematician who produced the first consistent and complete proof of the impossibility of not only trisecting the angle but also of doubling the cube and drawing regular polygons, was not seen for a century probably since he was not very well known and was overshadowed by other giants, and the credit of the proof was attributed to other names in the history of mathematics.<sup>44</sup>

Wantzel's proof of the problem of trisecting the angle is dated 1837. Salih Zeki, on the other hand, wrote his article in 1899 and, threw stones at Hüseyin Efendi's work of 1822 as well as to his contemporaries. In our turn, we can accuse Salih Zeki and his followers today for not knowing of Wantzel. But we are aware that the European mathematical community was also deaf to Wantzel's work for a century. Therefore, accusing Salih Zeki of not being aware of it is against the methods of the historiography of science. On the other hand, Montucla, to whom Salih Zeki refers in many of his other works, attributes the fame of the proof to James Gregory

<sup>43</sup> Richeson, Tales of Impossibility, 259.

<sup>44</sup> For a detailed research on the reasons of why Wantzel's work was not recognized for a century, see Jesper Lützen, "Why was Wantzel overlooked for a century? The Changing Importance of An Impossibility Result", *Historia Mathematica* 36 (2009): 374-394.

(d. 1675) in his famous history of mathematics, *Historie de Mathematiques*, not to Descartes. If we look at the later literature, we see that Descartes is not the only name, Gauss is also frequently mentioned. For example, the famous German mathematician Felix Klein (d. 1925), Salih Zeki's contemporary, credits Gauss with the impossibility proof of drawing all regular polygons in his work *Famous Problems of Elementary Geometry*.<sup>45</sup>

Wantzel did not gain the fame he deserved in the history of mathematics until 1913 when Florian Cajori (d. 1930) explained the situation in his famous book on the history of mathematics<sup>46</sup>. For example, Hamilton, whom Salih Zeki praised highly in his article, does not believe in Descartes' impossibility proofs as he noted in a letter to Augustus De Morgan (d. 1871) in 1852:

"Are you sure that it is impossible to trisect the angle by the Euclid? I have not to lament a single hour thrown away on the attempt, but fancy that it is rather a tact, a feeling, than a proof, which makes us think that the thing cannot be done. No doubt we are influenced by the cubic form of the algebraic equation. But would Gauss's inscription of the regular polygon of seventeen sides have seemed, a century ago, much less an impossible thing, by line and circle?"<sup>47</sup>

Even Hamilton was not aware of Wantzel's proof, or he also simply ignored them. As we understand from this letter, besides the discussion mentioned above, the ancient problems, and especially the problem of trisection continued to be discussed throughout the ninteenth century. In other words, the claim that Descartes settled the issue in the mid-seventeenth century - contrary to Salih Zeki's interpretation - was not generally accepted even at the end of the nineteenth century. In fact, with Wantzel, it became clear that this claim was not true.

# Exonerating Masdariyecizāde Hüseyin Efendi

In the light of the above discussions, it was seen that Salih Zeki accuses Hüseyin Efendi of not being aware of Descartes' proof of the impossibility of solving the problem. In fact, this claim is void since Descartes' proof was neither valid nor generally accepted.

<sup>45</sup> Felix Klein, Famous Problems of Elementary Geometry, (Boston: Ginn and Company, 1897), 16.

<sup>46</sup> Florian Cajori, A History of Mathematics, (London: Macmillan, 1893), 345.

<sup>47</sup> Richeson, Tales of Impossibility, 336.

The fact that Salih Zeki was not aware of the debates on the subject in Europe throughout the nineteenth century, and so, his regarding the solution as Descartes' achievement can be interpreted ironically; indeed, the situation he condemned Hüseyin Efendi, who lived seventy years before him, has hit himself. On the other hand, we should consider his ignorance of Wantzel and his proofs reasonable today, for we have seen that Wantzel's work was curiously ignored throughout a century. It is probable that this study came to the attention of Salih Zeki with its publication in the *Journal de Mathématiques pures et appliquées*, one of the most important publications in European mathematics at that time; however, it is not unlikely that this might not happened under the conditions of the period. However, we should not forget that he had opportunities Hüseyin Efendi did not have, such as an education in Europe, connections with mathematical circles there, and following European publications. In this respect, one can excuse Hüseyin Efendi's inability to follow contemporary debates, if not Descartes'. On the other hand, Salih Zeki bears more responsibility in this regard.

## Result

Masdariyecizāde Hüseyin Efendi's treatise *Teslīs-i Zāviye ve Kavs* deserves attention in the history of mathematics in the late Ottoman period as an attempt to a current debate in its own time and conditions. Our analysis of the contemporary literature on the subject reveals that Masdariyecizāde's work is evaluated only on the basis of Salih Zeki's series of articles written a century ago, which contains errors in terms of information and historical method. Since these evaluations are no more than repeating an already erroneous interpretation without verifying its accuracy by utilizing today's knowledge of the history of science, they lead to information pollution on the subject. Moreover, the fact that mathematical errors are made even when quoting Salih Zeki's article is important as it proves that technical expertise is essential in the history of science and the history of mathematics.

The reason why Masdariyecizāde's work does not provide a complete solution to the problem of trisecting the angle is explained by Salih Zeki in his series of articles. The important thing here is not whether the solution is correct or not. Today, we know that this problem cannot be solved with the desired method. In fact, it is well-known that extracting technical data from the history of science is often pointless and useless, considering the state of science today. The "misunderstanding" to which Masdariyecizāde's text is subjected is not technical

but historical. Moreover, since the proof of the unsolvability of the problem was not yet known in Masdariyecizāde's time, it is contrary to the methods of the history of science to accuse him of ignorance and or even to impute him concealing a known fact and to call his efforts "scandalous". The history of science is full of examples of failed attempts, and it is important to remember that these failures often open new doors in science. The methodological focus of historians of science should not be on showing the inaccuracy of the techniques but on determining what the effort in question corresponds to in its time, and its motives and effects.

We believe that Hüseyin Efendi's work should be reconsidered in accordance with the method of the history of science, with the conditions of the time, but without making the mistake of judging him from a hundred-year ahead perspective as Salih Zeki did with him. On the other hand, while the sources that Salih Zeki could not obtain and the information that was produced after him are within our reach, we think that it is a necessity of the historiography of science to confirm what Salih Zeki said and to follow the progress of story until today.

In relation to this, the question that seriously concerns today's academic world is that if researchers accept a popular publication written according to the research methods and writing fashion of a hundred years ago as the sole source for their academic studies, and present an article of a hundred years ago to the academic world as a research result, without investigating the accuracy of what is said, without reviewing the literature that has been created in a century, and journal referees and editors do not question this, what academic development can be mentioned? Leave aside the technical answers to this question, in this case, the details of which we have examined is also interesting in that they provide an insight to the analyzes on the Ottoman intellectual's view of himself and his predecessors through the example of Salih Zeki, and the Turkish researchers' view of the Ottomans through the interpretations in contemporary literature.

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